# Convolution:

$$\boxed{f(t) * g(t) = \int_0^t f(\tau)\, g(t-\tau)\, d\tau}$$

Suppose, $f(t) = e^{-t}$ and $g(t) = \sin t$

$\cos(t - \cancel{t}) = \cancel{t}$ $-\cos(t - \cancel{0})$

$= I$ ✓

$$f(t) * g(t) = \int_0^t e^{-\tau} \sin(t-\tau)\, d\tau$$

$\tau \to t$ , $e^{-t}$ $- \cancel{e^{-0}}$ $1$

I by Part

$u = e^{-\tau} \to du = -e^{-\tau} dt$

$dv = \sin(t-\tau)\, d\tau$

$v = -\cos(t-\tau)$

$= \cos(t-\tau)$

$$I = e^{-\tau} \cos(t-\tau)\Big|_0^t + \int_0^t e^{-\tau} \cos(t-\tau)\, d\tau$$

$$I = e^{-\tau} \cos(t-\tau)\Big|_0^t - e^{-\tau} \sin(t-\tau)\Big|_0^t - \cancel{\int_0^t e^{-\tau} \sin(t-\tau)\, d\tau}$$

$$I = +e^{-t} - \cos(t) - e^{-t} + \sin(t) - I$$

$$2I = e^{-t} - \cos t + \sin t$$

$$I = \frac{1}{2}\left(e^{-t} - \cos t + \sin t\right)$$

$u = e^{-\tau} \to du = -e^{-\tau} d\tau$

① ②

$dv = \cos(t-\tau) \to v = -\sin(t-\tau)$

$u = e^{-\tau} \to du = -e^{-\tau}$

$dv = \cos(t-\tau) \to v = -\sin(t-\tau)$

$\int u\, dv = uv - \int v\, du$

# Q1:

Convolution

$$f(t) * g(t) = \int_0^t f(\tau)\, g(t-\tau)\, d\tau$$

Suppose $f(t) = e^{-t}$ and $g(t) = \sin t$

$$f(t) * g(t) = \int_0^t e^{-\tau} \sin(t-\tau)\, d\tau$$

$$u = e^{-\tau} \Rightarrow du = -e^{-\tau}\, d\tau$$

$$dv = \sin(t-\tau)\, d\tau \Rightarrow v = -\cos(t-\tau)$$

$$I = e^{-\tau}\cos(t-\tau)\Big|_0^t + \int_0^t e^{-\tau}\cos(t-\tau)\, d\tau$$

$$u = e^{-\tau} \Rightarrow du = -e^{-\tau}\, d\tau$$

$$dv = \cos(t-\tau)\, d\tau \Rightarrow v = -\sin(t-\tau)$$

$$I = e^{-\tau}\cos(t-\tau)\Big|_0^t - e^{-\tau}\sin(t-\tau)\Big|_0^t - \int_0^t e^{-\tau}\sin(t-\tau)\, d\tau$$

$$I = e^{-t} - \cos(t) - \cancel{e^{-t\,0}} + \sin(t) - I$$

$$2I = e^{-t} - \cos t + \sin t$$

$$I = \frac{1}{2}\left(e^{-t} - \cos t + \sin t\right)$$

If, $f(x) = e^{t}$ , $g(t) = \cos t$

$$f(t) * g(t) = \int_0^t f(\tau)\, g(t-\tau)\, d\tau = I$$

$$= \int_0^t \underbrace{e^{\tau}}_{u} \underbrace{\cos(t-\tau)\, d\tau}_{dv}$$ Integration by Parts

$u = e^{\tau} \longrightarrow du = e^{\tau}$

$\int dv = \cos(t-\tau) \longrightarrow v = -\sin(t-\tau)$

$$I = -e^{\tau}\sin(t-\tau)\Big|_0^t + \int_0^t \underbrace{e^{\tau}}_{u} \underbrace{\sin(t-\tau)\, d\tau}_{dv}$$

$u = e^{\tau} \longrightarrow du = e^{\tau}$

$\int dv = \sin(t-\tau) \longrightarrow v = \cos(t-\tau)$

$$I = \underbrace{-e^{\tau}\sin(t-\tau)\Big|_0^t}_{=0} + e^{\tau}\cos(t-\tau)\Big|_0^t + \underbrace{\int_0^t e^{\tau}\cos(t-\tau)\, dt}_{I}$$

$$\boxed{\int u\, dv = uv - \int v\, du}$$

# Fourier Transformer (F.T):

$g(t) \longleftrightarrow G(f)$

$$G(f) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt$$

unit step → 1

Ex, If $g(t) = e^{-\cancel{a}t}\, u(t)$, find $G(F)$

$$G(F) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt = \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, dt$$

$e^{-at - j\omega t} \rightarrow (-a - j\omega)t$

$-(a+j\omega)t$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt = G(f) = \frac{e^{-(\cancel{a+j\omega})t}}{-(a+j\omega)} \Bigg|_{0}^{\infty} = \frac{\cancel{0} - 1}{\cancel{-}(a+j\omega)}$$

$-e^{\cancel{0}}$

$$= \boxed{\frac{1}{3\,\cancel{a} + j2\pi f}}$$

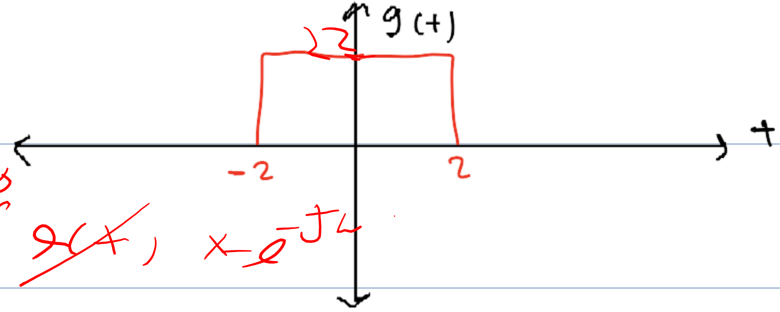
$\boxed{\omega = 2\pi f}$

If $g_1(t) \longleftrightarrow G_1(f)$ & $g_2(t) \longleftrightarrow G_2(f)$

then, $a_1 g_1(t) + a_2 g_2(t) \longleftrightarrow a_1 G_1(f) + a_2 G_2(f)$

→ Linear transforms

Ex,

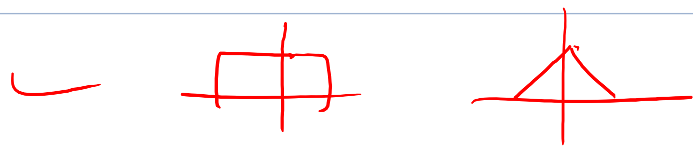


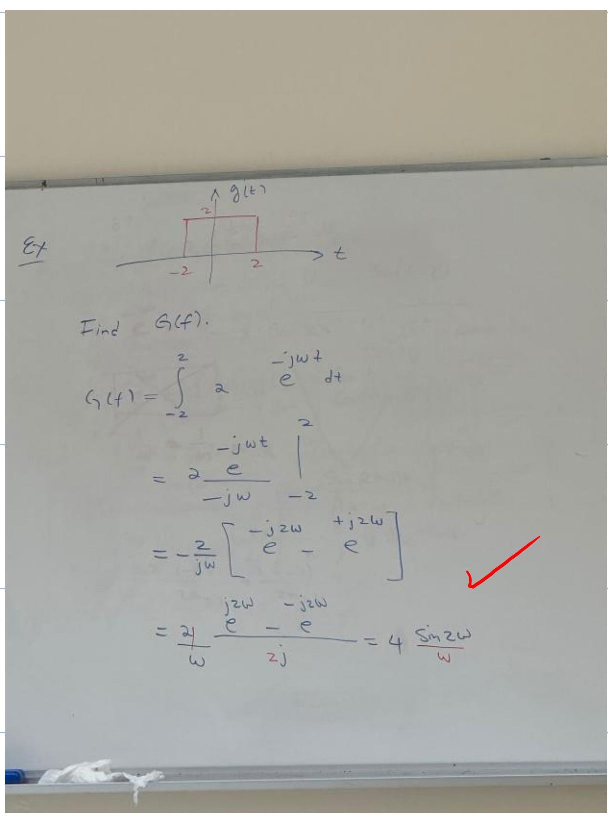

Find $G_1(f)$: $\int_{-\infty}^{\infty} \cancel{g(t)}, \times e^{-j\omega}$

$$G_1(f) = \int_{-2}^{2} 2\, e^{-j\omega t}\, dt = 2\, \frac{e^{-j\omega t}}{-j\omega}\Bigg|_{-2}^{2}$$

$$= \frac{2\times 2}{-j\omega 2}\left[ e^{-j2\omega} - e^{+j2\omega} \right]$$

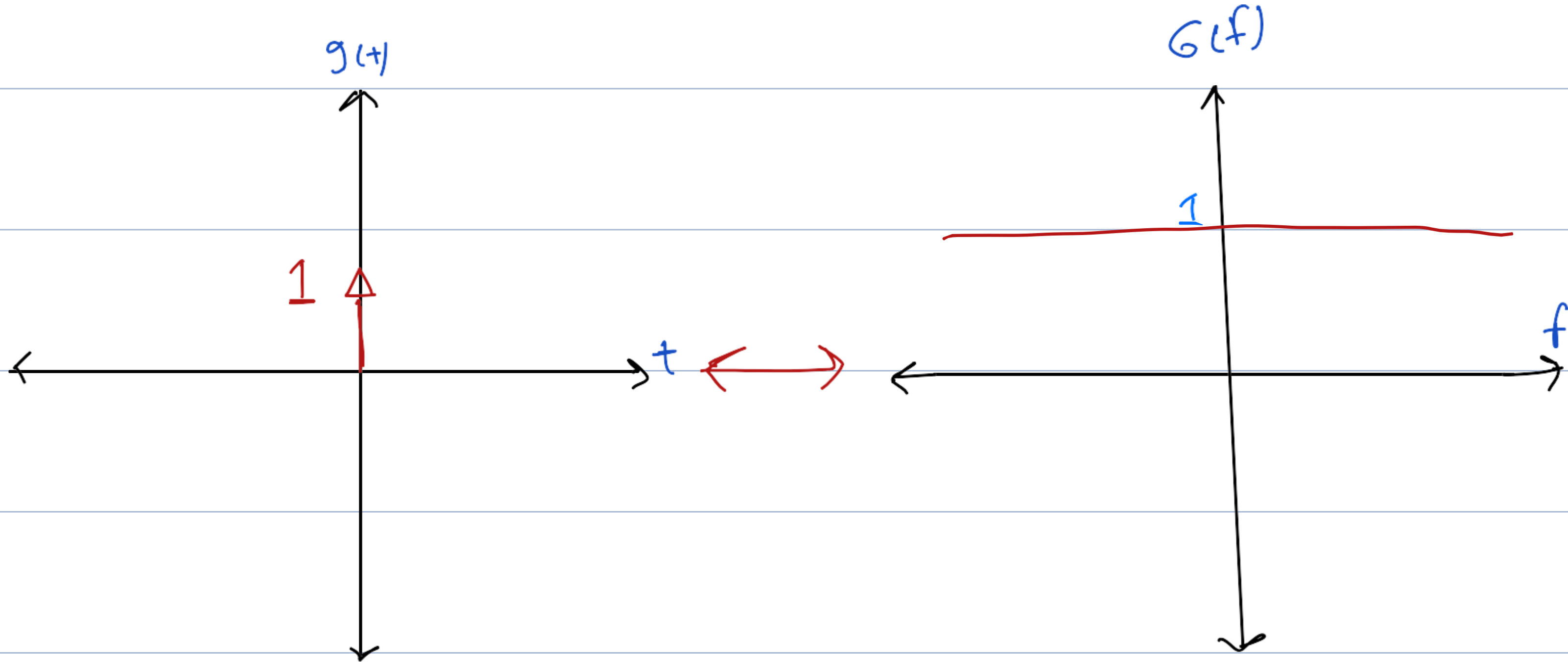
$$= \frac{2\times 2}{\omega}\left[ \frac{e^{j2\omega} - e^{-j2\omega}}{2j} \right] = \frac{4\sin 2\omega}{\omega} = 8\,\mathrm{Sinc}(2\omega)$$



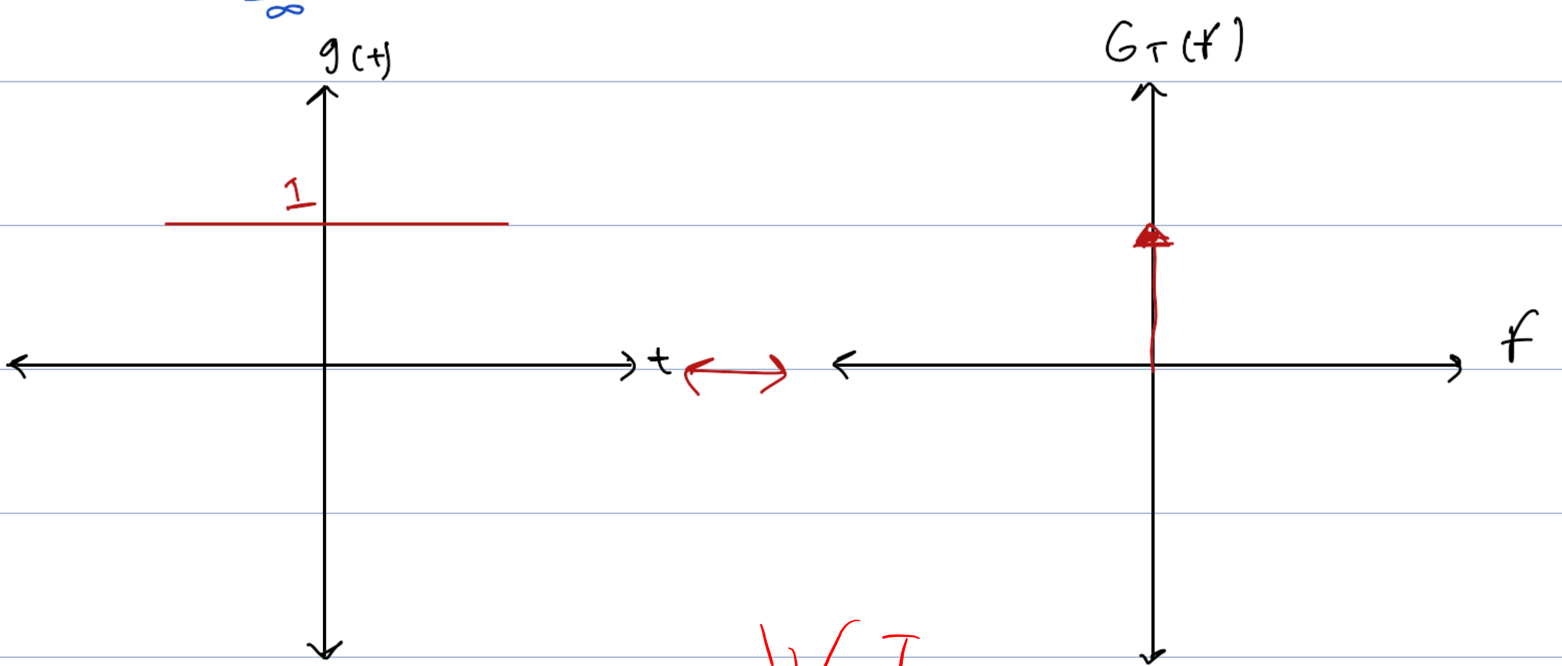



$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$




$$G_T(f) = \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\, dt = e^{-j\omega(0)} = \underline{1}$$



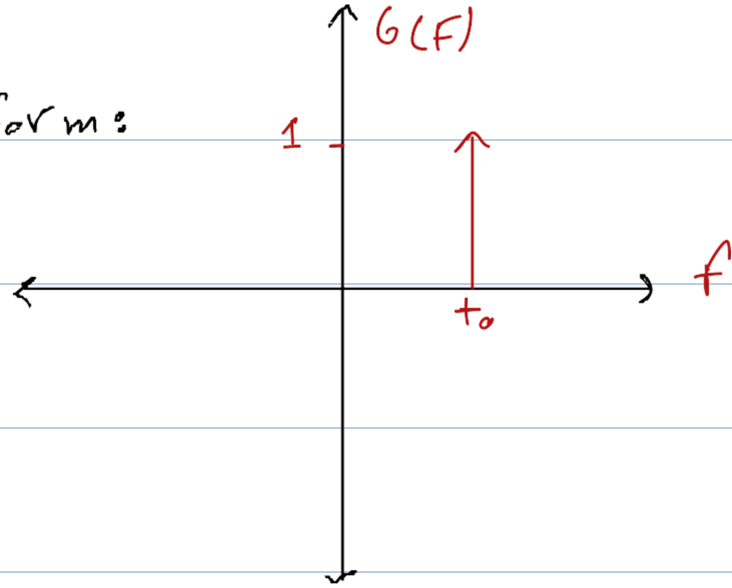

UFT

$$\delta(t) \to \delta(f) = 1$$

$$FT^{-1}(1) \to \underline{\delta(t)}$$

$$\boxed{\begin{array}{l} \delta(t) \longleftrightarrow 1 \\ 1 \longleftrightarrow \delta(t) \end{array}}$$

Find the impulse Fouries Transform:

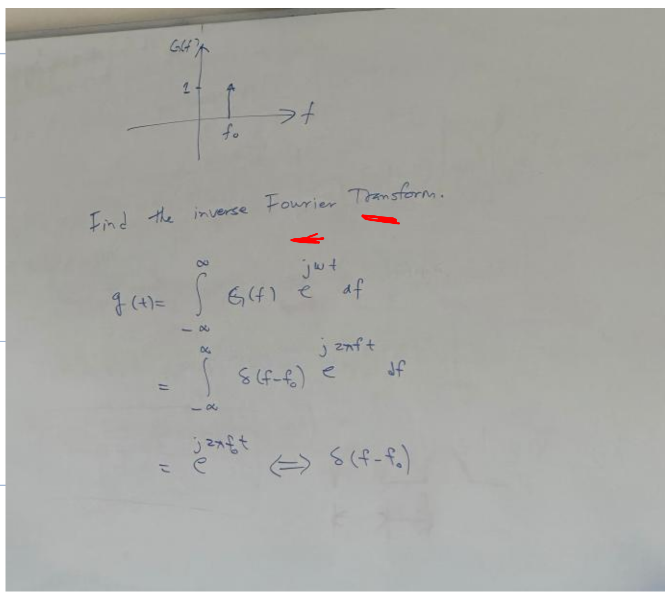



$\delta(F)$

$$g(t) = \int_{-\infty}^{\infty} G_1(f)\, e^{j\omega t}\, df$$

$$= \int_{-\infty}^{\infty} \delta(f-f_0)\, e^{j2\pi f t}\, df = e^{j2\pi f_0 t}$$

$$= e^{j2\pi f_0 t} \longleftrightarrow \delta(f-f_0)$$




1

If, $g(t) = \cos W_0 t$ , Find (F.T)?

$$G_1(f) = \int_{-\infty}^{\infty} \cos W_0 t \; e^{-jWt} \, dt$$

$$= \int_{-\infty}^{\infty} \left( \frac{e^{jW_0 t} + e^{-jW_0 t}}{2} \right) e^{-jWt} \, dt$$

$e^{jw_0 t - Jwt}$

$= e^{(Jw_0 - Jw)t}$

$e^{Jt(w_0 - w)}$

$$= \frac{1}{2} \int_{-\infty}^{\infty} \left( e^{-j(W - W_0)t} + e^{-j(W + W_0)t} \right) dt$$

$e^{-Jt(W - w_0)}$

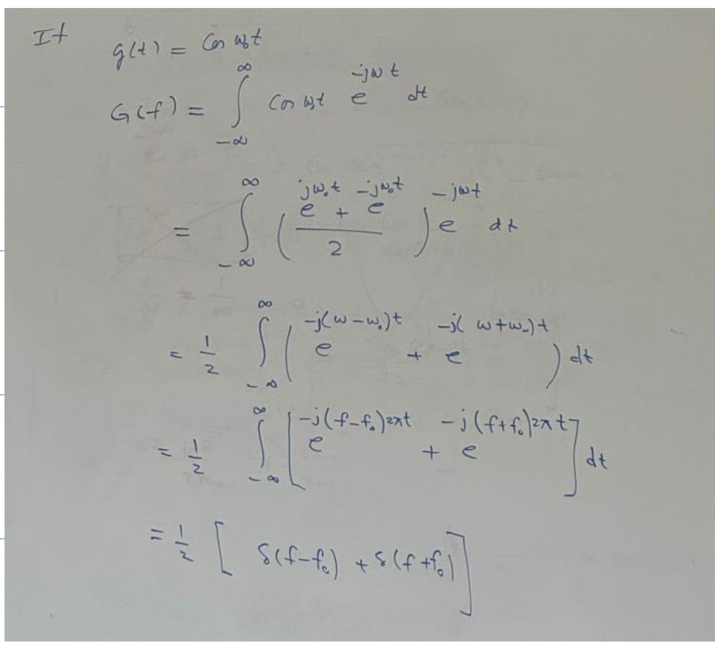
$2\pi \quad 2\pi \quad 2\pi$

$$= \frac{1}{2} \int_{-\infty}^{\infty} \left[ e^{-j(f - f_0) 2\pi t} + e^{-j(f + f_0) 2\pi t} \right] dt$$

$w = 2\pi f$

$w_0 = 2\pi f_0$

$$= \frac{1}{2} \left[ \delta(f - f_0) + \delta(f + f_0) \right]$$



If

$g(t) = \cos w_0 t$

$$G(f) = \int_{-\infty}^{\infty} \cos w_0 t \; e^{-jwt} \, dt$$

$$= \int_{-\infty}^{\infty} \left( \frac{e^{jw_0 t} + e^{-jw_0 t}}{2} \right) e^{-jwt} \, dt$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} \left( e^{-j(w - w_0)t} + e^{-j(w + w_0)t} \right) dt$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} \left[ e^{-j(f - f_0) 2\pi t} + e^{-j(f + f_0) 2\pi t} \right] dt$$

$$= \frac{1}{2} \left[ \delta(f - f_0) + \delta(f + f_0) \right]$$